Preprint gr-qc/9905024
April 1999
revised August 1999

Energy Conservation and the Unruh Effect

Riuji Mochizuki[1†] **and Takayuki Suga**[2‡]

[1] *Laboratory of Physics, Tokyo Dental College*
1-2-2 Masago, Mihama-ku, Chiba 261-8502, Japan

[2] *Graduate School of Science and Technology, Chiba University*
Yayoi, Inage-ku Chiba 263-8522, Japan

[†] E-mail: rjmochi@tdc.ac.jp

[‡] E-mail: psuga@cuphd.nd.chiba-u.ac.jp

Abstract

In this paper it is explicitly demonstrated that the energy conservation law is kept when a detector uniformly accelerated in the Minkowski vacuum is excited and emits a particle. This fact had been hidden in conventional approaches in which detectors were considered to be forced on trajectories. To lift the veil we suggest a detector model written in terms of the Minkowski coordinates. In this model the Hamiltonian of the detector involves a classical potential term instead of the detector's fixed trajectory. The transition rate agrees with the corresponding conventional one in the limit of an infinite mass detector though even then the recoil remains.

1 Introduction

It is more than a quarter of a century that has passed since his vital paper was given by Fulling [1]. He pointed out that an inertial observer and a uniformly accelerated one would construct the operator algebras which are not unitarily equivalent to each other. The Bogoliubov transformation between them demonstrates a well-known effect which is often called the Unruh effect [2, 3]. If the Minkowski vacuum is described by a uniformly accelerated observer, it is not a state in which there are no particles but a thermal bath (the so-called Fulling-Davies-Unruh thermal bath) which is characterized by a temperature proportional to his proper acceleration. This observation carries a serious question of the particle concept and necessity of preparing measuring devices. DeWitt [4] provided a point-like detector model with two internal energy levels which is coupled linearly to a scalar field. Using this model, the Unruh effect consequentially contains a paradoxical event: a DeWitt detector uniformly accelerated in the Minkowski vacuum can be excited and emit the radiation. Many authors seeked a source of this mysterious energy. For example, Unruh and Wald [5] and Takagi [6] casted their consideration over vacuum fluctuation of the field. Birrell and Davies [7] suspected that extra work done by the external force which accelerated the detector was the source of this energy. In their approach, however, the detector's trajectory is fixed. This curtains energy and momentum conservations. Though Parentani [8] treated the detector's trajectory as a dynamical variable, a description of the flow on energy was not given.

The main purpose of this paper is to clarify the fact that the energy of the radiation and difference of the detector's rest mass comes directly from the kinetic energy via the recoil of the detector. In other words, proved in this paper is that the Unruh effect is a phenomenon satisfying the energy conservation law. To this end, the conventional DeWitt detector model is modified into a form in which the detector is not fixed on any trajectories. Instead, the Hamiltonian of the detector involves a classical potential term if it is considered to be accelerated. The rest mass of the detector treated as finite throughout calculation, the transition rate obtained using this model agrees with conventional one after taking the limit of the infinite mass detector. In our calculation it is essential that the translational invariance is broken due to the potential energy of the detector.

This paper is organized as follows. In the second section the transition rate of a detector in inertial motion is discussed to demonstrate difference between this case and that of the accelerated detector. In the third section the transition rate corresponding to the Fulling-Davies-Unruh thermal bath is recreated as that of the detector moving in classical potential with a constant gradient. In the last section we summarize the result and make a remark.

2 Detector model with no fixed trajectory

A detector which is coupled with a scalar field via a monopole interaction is called a DeWitt detector. In the conventional DeWitt detector model, the detector is supposed to move along a trajectory $x^\mu(\tau)$, where τ is the detector's proper time. The interaction Hamiltonian H_I is

$$H_I = \int d^3\mathbf{x} \int_{-\infty}^{\infty} d\tau c_0 m(\tau)\phi(x)\delta^{(4)}(x - x(\tau)), \tag{2.1}$$

where c_0 is a coupling constant and ϕ is a scalar field which interacts with the detector. The detector's monopole moment operator $m(\tau)$ is written in the interaction picture as

$$m(\tau) = \mathrm{e}^{iH_0\tau} m(0) \mathrm{e}^{-iH_0\tau}, \tag{2.2}$$

$$m(0) =\mid m >< m_0 \mid, \tag{2.3}$$

where m and m_0 are the rest masses on the detector's upper and lower energy levels, respectively. The free Hamiltonian H_0 is defined on the detector's trajectory, so that

$$H_0 \mid m >= m \mid m > . \tag{2.4}$$

The amplitude for the transition in which a detector is excited and simultaneously emits a scalar particle with its momentum $\mathbf{k}$ in Minkowski vacuum $\mid 0_M >$ is given by first order perturbation theory as

$$\begin{aligned} A &= -i < m, 1_{\mathbf{k}} \mid \int d^4x \int_{-\infty}^{\infty} d\tau c_0 m(\tau)\phi(x)\delta^{(4)}(x - x(\tau)) \mid 0_M, m_0 > \\ & -i < m, 1_{\mathbf{k}} \mid \int_{-\infty}^{\infty} c_0 m(\tau)\phi[x(\tau)] d\tau \mid 0_M, m_0 > . \end{aligned} \tag{2.5}$$

ϕ is expanded in terms of Minkowski plane wave modes, thus

$$< 1_{\mathbf{k}} \mid \phi(x) \mid 0_M >= \frac{n}{\sqrt{\omega}} \mathrm{e}^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}, \tag{2.6}$$

where n is a normalization constant and $\omega^2 =\mid \mathbf{k} \mid^2$ for the real particle. Substituting (2.2), (2.3), (2.4) and (2.6) into (2.5) yields

$$A = -i\frac{nc_0}{\sqrt{\omega}} \int_{-\infty}^{\infty} d\tau \mathrm{e}^{i\Delta m\tau + i(\omega t(\tau) - \mathbf{k}\cdot\mathbf{x}(\tau))}, \tag{2.7}$$

where

$$\Delta m = m - m_0.$$

To discuss physical significance of the different vacua and energy conservation of the Unruh effect, detector models should be examined carefully. In the conventional DeWitt model, as noted above, a trajectory of the detector is imposed and the coordinates in the transition amplitudes are treated as functions of the detector's proper time. This is a dynamically rather misty manipulation. Hence we suggest a modified DeWitt detector model in which the detector moves in classical potential but is not forced on any trajectories.

First, we consider the case in which the detector moves inertially. We introduce the detector fields Φ_0 and Φ with rest masses m_0 and m, respetively. They satisfy the Klein-Gordon equation

$$(\Box + {m_0}^2)\Phi_0(x) = 0,$$

$$(\Box + m^2)\Phi(x) = 0$$

and are expanded in complete sets of solutions of the above equations:

$$\Phi_0 \quad (x) = \Phi_0^+(x) + \Phi_0^-(x) \tag{2.8}$$

$$\Phi_0^+(x) = \int d^3\mathbf{P}_0 \frac{N_0}{\sqrt{E_0}} a_0(\mathbf{P}_0) \mathrm{e}^{-i(E_0 t - \mathbf{P}_0 \cdot \mathbf{x})} \tag{2.9}$$

$$\Phi_0^-(x) = \int d^3\mathbf{P}_0 \frac{N_0}{\sqrt{E_0}} {a_0}^\dagger(\mathbf{P}_0) \mathrm{e}^{+i(E_0 t - \mathbf{P}_0 \cdot \mathbf{x})}, \tag{2.10}$$

$$\Phi \quad (x) = \Phi^+(x) + \Phi^-(x) \tag{2.11}$$

$$\Phi^+(x) = \int d^3\mathbf{P} \frac{N}{\sqrt{E}} a(\mathbf{P}) \mathrm{e}^{-i(Et - \mathbf{P} \cdot \mathbf{x})} \tag{2.12}$$

$$\Phi^-(x) = \int d^3\mathbf{P} \frac{N}{\sqrt{E}} a^\dagger(\mathbf{P}) \mathrm{e}^{+i(Et - \mathbf{P} \cdot \mathbf{x})}, \tag{2.13}$$

$$E_0 = ({\mathbf{P}_0}^2 + {m_0}^2)^{1/2}, \quad E = (\mathbf{P}^2 + m^2)^{1/2}, \tag{2.14}$$

where N_0 and N are normalization constants and a_0 (${a_0}^\dagger$) and a ($a^\dagger$) are annihilation (creation) operators of the detector with rest masses m_0 and m, respectively.

In terms of these fields, the interaction Hamiltonian density $\mathcal{H}_\mathrm{I}$ is defined as

$$\mathcal{H}_\mathrm{I} = c\Phi^-(x)\Phi_0^+(x)\phi(x), \tag{2.15}$$

where c is a coupling constant. The transition amplitude corresponding to (2.5) is

$$A = -i < \mathbf{P}, m \mid < 1_\mathbf{k} \mid c \int d^4x \Phi^-(x)\Phi_0^+(x)\phi(x) \mid 0_M > \mid m_0, \mathbf{P}_0 >, \tag{2.16}$$

where

$$\mid m, \mathbf{P} >= a^\dagger(\mathbf{P}) \mid 0 >,$$
$$\mid m_0, \mathbf{P}_0 >= {a_0}^\dagger(\mathbf{P}_0) \mid 0 > .$$

Using the expansions (2.9) and (2.13), this amplitude becomes

$$A \sim \frac{1}{\sqrt{EE_0\omega}} \int d^4x \mathrm{e}^{i(E - E_0 + \omega)t - i(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}) \cdot \mathbf{x}} \tag{2.17}$$

$$\sim \frac{1}{\sqrt{EE_0\omega}} \delta(E - E_0 + \omega)\delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}). \tag{2.18}$$

These delta functions explicitly indicate energy and momentum conservations at the interaction between the detector and the field ϕ. The amplitude (2.18) always vanishes because the arguments of these delta functions cannot be zero simultaneously [9]. Indeed, when the momentum conservation is used, the argument of the first delta function becomes

$$\begin{aligned} E - E_0 + \omega &= m\gamma - m_0\gamma_0 + \omega \\ &= {\gamma_0}^{-1}(m\tilde{\gamma} - m_0) + (m\gamma\mathbf{V} - m_0\gamma_0\mathbf{V}_0) \cdot \mathbf{V}_0 + \omega \\ &= {\gamma_0}^{-1}(m\tilde{\gamma} - m_0) + (\omega - \mathbf{k} \cdot \mathbf{V}_0), \end{aligned} \tag{2.19}$$

where $\mathbf{V}_0 = \mathbf{V}_0(\mathbf{P}_0)$ and $\mathbf{V} = \mathbf{V}(\mathbf{P})$ are velocities of the detector before and after the interaction, respectively, and the γ factors are defined as

$$\gamma_0 = (1 - \mid \mathbf{V}_0 \mid^2)^{-1/2},$$

$$\gamma = (1 - \mid \mathbf{V} \mid^2)^{-1/2},$$

$$\tilde{\gamma} = (1 - \mid \tilde{\mathbf{V}} \mid^2)^{-1/2} = \gamma\gamma_0(1 - \mathbf{V}\cdot\mathbf{V}_0), @ \quad (\tilde{\mathbf{V}} : \text{relative velocity}).$$

Recalling $m > m_0$, $\tilde{\gamma} > 1$ and $\omega > \mathbf{k}\cdot\mathbf{V}_0$, the right-hand side of (2.19) is always positive. To argue its correspondence with the conventional DeWitt model, we go back to the amplitude (2.17). Perfoming the $\mathbf{x}$ integration and using (2.19), this becomes

$$\begin{aligned} A &\sim \frac{1}{\sqrt{EE_0\omega}} \int dt \mathrm{e}^{i(m\gamma(\mathbf{P}_0,\mathbf{k}) - m_0\gamma_0(\mathbf{P}_0)+\omega)t} \delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}) \\ &\sim \frac{1}{\sqrt{EE_0\omega}} \int dt \mathrm{e}^{i({\gamma_0}^{-1}(m\tilde{\gamma} - m_0) + (\omega - \mathbf{k}\cdot\mathbf{V}_0))t} \delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}). \end{aligned} \tag{2.20}$$

In the limit of an infinite mass detector($m \to \infty$, $m_0 \to \infty$ while maintaining Δm finite,

$$\mathbf{V}_0 = \mathbf{V}, \quad \tilde{\mathbf{V}} = 0.$$

Then (2.20) becomes

$$A \sim \frac{1}{\sqrt{\omega}} \int d\tau \mathrm{e}^{i\Delta m\tau + i(\omega t(\tau) - \mathbf{k}\cdot\mathbf{x}(\tau))} \delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}), \tag{2.21}$$

where τ is the detector's proper time, and $t(\tau)$ and $\mathbf{x}(\tau)$ are defined as

$$t(\tau) = \gamma_0\tau,$$

$$\mathbf{x}(\tau) = \mathbf{V}_0 t(\tau).$$

What is considered is the transition in which the rest mass of the detector undergoes the change $m_0 \to m$ irrespective of the final momentum of the detector. Hence the transition rate R is

$$\begin{aligned} R &\sim \int d^3\mathbf{P} \int d^3\mathbf{k} \mid A \mid^2 \\ &\sim \int d^3\mathbf{P} \int d^3\mathbf{k} \frac{1}{\omega} \int d\overline{\tau} \int d(\Delta\tau) \mathrm{e}^{i\Delta m\Delta\tau + i(\omega t(\Delta\tau) - \mathbf{k}\cdot\mathbf{x}(\Delta\tau))} \left[\delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k})\right]^2 \\ &\sim \int d^3\mathbf{k} \frac{1}{\omega} \int d\overline{\tau} \int d(\Delta\tau) \mathrm{e}^{i\Delta m\Delta\tau + i(\omega t(\Delta\tau) - \mathbf{k}\cdot\mathbf{x}(\Delta\tau))}, \end{aligned} \tag{2.22}$$

where

$$\overline{\tau} = \frac{1}{2}(\tau + \tau'),$$

$$\Delta\tau = \tau - \tau'$$

and we have used

$$\left[\delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k})\right]^2 \sim \delta^{(3)}(\mathbf{P} - \mathbf{P}_0 + \mathbf{k}).$$

This agrees with that obtained using the conventional DeWitt model up to a constant factor. Performing the $\Delta\tau$ integration, we obtain

$$R \sim \delta(\Delta m + \gamma_0(\omega - \mathbf{k}\cdot\mathbf{V}_0)) = 0. \tag{2.23}$$

This transition rate for the inertial detector always vanishes. Note that it is not due to energy conservation but due to the fact that energy and momentum conservations are never concomitant for $\Delta m \geq 0$.

3 the Unruh effect

In this section we demonstrate the Unruh effect as response of a detector moving in a classical scalar potential $-Fz$ in which F is a constant. The Klein-Gordon equations the detector fields $\Phi_0{}'$ and Φ' should satisfy are

$$\Big[(i\frac{\partial}{\partial t} + Fz)^2 + \nabla^2 - {m_0}^2\Big]\Phi_0' = 0,$$

$$\Big[(i\frac{\partial}{\partial t} + Fz)^2 + \nabla^2 - m\ ^2\Big]\Phi' = 0.$$

We can obtain the solutions of these equations by the aid of WKB approximation method. In the classically allowed region, the detector's fields are expanded as

$$\begin{aligned}
\Phi_0'(x) &= \Phi_0'^{+}(x) + \Phi_0'^{-}(x) && (3.1)\\
\Phi_0'^{+}(x) &= \int_0^\infty dE_0 \int_{-\infty}^{\infty} dP_{0x} \int_{-\infty}^{\infty} dP_{0y} \frac{N_0'}{\sqrt{\mid P_{0z}\mid}} a_0(E_0, P_{0x}, P_{0y}) \\
&\quad \times \mathrm{e}^{-i(E_0 t - P_{0x}x - P_{0y}y - \int^z P_{0z}dz)}\theta\Big(z - \frac{1}{F}(E_0 - \sqrt{{m_0}^2 + P_{0x}^2 + P_{0y}^2})\Big) && (3.2)\\
\Phi_0'^{-}(x) &= \int_0^\infty dE_0 \int_{-\infty}^{\infty} dP_{0x} \int_{-\infty}^{\infty} dP_{0y} \frac{N_0'}{\sqrt{\mid P_{0z}\mid}} {a_0}^\dagger(E_0, P_{0x}, P_{0y}) \\
&\quad \times \mathrm{e}^{+i(E_0 t - P_{0x}x - P_{0y}y - \int^z P_{0z}dz)}\theta\Big(z - \frac{1}{F}(E_0 - \sqrt{{m_0}^2 + P_{0x}^2 + P_{0y}^2})\Big), && (3.3)
\end{aligned}$$

$$\begin{aligned}
\Phi'(x) &= \Phi'^{+}(x) + \Phi'^{-}(x) && (3.4)\\
\Phi'^{+}(x) &= \int_0^\infty dE \int_{-\infty}^{\infty} dP_x \int_{-\infty}^{\infty} dP_y \frac{N'}{\sqrt{\mid P_z\mid}} a(E, P_x, P_y) \\
&\quad \times \mathrm{e}^{-i(Et - P_x x - P_y y - \int^z P_z dz)}\theta\Big(z - \frac{1}{F}(E - \sqrt{m^2 + {P_x}^2 + {P_y}^2})\Big) && (3.5)\\
\Phi'^{-}(x) &= \int_0^\infty dE \int_{-\infty}^{\infty} dP_x \int_{-\infty}^{\infty} dP_y \frac{N'}{\sqrt{\mid P_z\mid}} a^\dagger(E, P_x, P_y) \\
&\quad \times \mathrm{e}^{+i(Et - P_x x - P_y y - \int^z P_z dz)}\theta\Big(z - \frac{1}{F}(E - \sqrt{m^2 + {P_x}^2 + {P_y}^2})\Big), && (3.6)
\end{aligned}$$

where the mode functions have been selected to become plane wave when $F \to 0$ and

$$| P_{0z} |= \sqrt{(E+Fz)^2 - {m_0}^2 - P_{0x}^2 - P_{0y}^2}, \tag{3.7}$$

$$| P_z |= \sqrt{(E+Fz)^2 - m^2 - P_x^2 - P_y^2}. \tag{3.8}$$

The transition amplitude in this case is

$$A = -i < P_y, P_x, E, m |< 1_{\mathbf{k}} | c \int d^4x \Phi'^{-}(x) \Phi_0'^{+}(x) \phi(x) | 0_M >| m_0, E_0, P_{0x}, P_{0y} >, \tag{3.9}$$

where

$$| m, E, P_x, P_y >= a^\dagger(E, P_x, P_y) | 0 >,$$

$$| m_0, E_0, P_{0x}, P_{0y} >= {a_0}^\dagger(E_0, P_{0x}, P_{0y}) | 0 > .$$

Using the expansions (3.2) and (3.6), this amplitude becomes

$$\begin{aligned} A \;\sim\; & \int d^4x \frac{1}{\sqrt{| P_z || P_{0z} | \omega}} \mathrm{e}^{i(E-E_0+\omega)t - i(P_x - P_{0x} + k_x)x - i(P_y - P_{0y} + k_y)y} \mathrm{e}^{-i(\int^z P_z dz - \int^z P_{0z} + k_z z)} \\ & \times \theta\Big(z - \frac{1}{F}(E - \sqrt{m^2 + {P_x}^2 + {P_y}^2})\Big) \theta\Big(z - \frac{1}{F}(E_0 - \sqrt{{m_0}^2 + P_{0x}^2 + P_{0y}^2})\Big) \\ \sim\; & \int dz \frac{1}{\sqrt{| P_z || P_{0z} | \omega}} \mathrm{e}^{-i(\int^z P_z dz - \int^z P_{0z} + k_z z)} \\ & \times \delta(E - E_0 + \omega) \delta(P_x - P_{0x} + k_x) \delta(P_y - P_{0y} + k_y) \\ & \times \theta\Big(z - \frac{1}{F}(E - \sqrt{m^2 + {P_x}^2 + {P_y}^2})\Big) \theta\Big(z - \frac{1}{F}(E_0 - \sqrt{{m_0}^2 + P_{0x}^2 + P_{0y}^2})\Big). \end{aligned} \tag{3.10}$$

The first delta function in this equation explicitly indicates energy conservation, that is, the energy paid for the radiation comes from the detector's kinetic energy. This statement may give a somewhat peculiar impression if minding a fact that a slowly moving detector does not have enough kinetic energy to satisfy the above equation. This is, however, only alarmism because there is no inertial frame in which the detector is always at rest and because the time when the interaction occurs cannot be exactly determined due to the uncertinly principle.

To see correspondence with the conventional approach, we limit the detector's motion on z-axis:

$$P_{0x} = P_{0y} = 0.$$

Then the transition amplitude (3.10) becomes

$$\begin{aligned} A \;\sim\; & \int dz \frac{1}{\sqrt{| P_z || P_{0z} | \omega}} \delta(m\gamma - m_0\gamma_0 + \omega) \delta(P_x + k_x) \delta(P_y + k_y) \mathrm{e}^{-i \int^z (m\gamma V_z - m_0 \gamma_0 V_0 + k_z) dz} \\ & \times \theta\Big(z - \frac{1}{F}(E - \sqrt{m^2 + {P_x}^2 + {P_y}^2})\Big) \theta\Big(z - \frac{1}{F}(E_0 - m_0)\Big), \end{aligned} \tag{3.11}$$

where

$$
\begin{aligned}
\gamma &= \frac{E+Fz}{m}, \\
\gamma_0 &= \frac{E_0+Fz}{m_0}, \\
\mid V_z \mid &= \frac{\sqrt{(E+Fz)^2-m^2}}{E+Fz}, \\
\mid V_{0z} \mid &= \frac{\sqrt{(E_0+Fz)^2-{m_0}^2}}{E_0+Fz}.
\end{aligned}
$$

Substituting (2.19) into the exponent in (3.11) yields

$$
m\gamma V_z - m_0\gamma_0 V_{0z} + k_z = -\frac{\omega}{V_{0z}} + k_z - \frac{m\tilde{\gamma}-m_0}{\gamma_0 V_{0z}}.
$$

In the limit of the infinite mass detector, (3.11) becomes

$$
\begin{aligned}
A \sim \quad & \int_{-\frac{E_0-m_0}{F}}^{\infty} \frac{dz}{\sqrt{\mid P_z \mid\mid P_{0z} \mid \omega}} \delta(m\gamma - m_0\gamma_0 + \omega)\delta(P_x)\delta(P_y) \\
& \times \exp\Big[-i\int^z \Big(-\frac{\omega}{V_{0z}} + k_z - \frac{m\tilde{\gamma}-m_0}{\gamma_0 V_{0z}}\Big)dz\Big] \\
\sim \quad & \int_{-\infty}^{\infty} \frac{d\tau}{\sqrt{\omega}} \delta(m\gamma - m_0\gamma_0 + \omega)\delta(P_x)\delta(P_y) \\
& \times \exp\Big[+i\Big(\Delta m\tau + \omega t(\tau) - k_z z(\tau)\Big)\Big],
\end{aligned} \tag{3.12}
$$

where

$$
z(\tau) + \frac{E_0}{F} = \frac{m_0}{F}\cosh\frac{F\tau}{m_0}, \tag{3.13}
$$

$$
t(\tau) = \frac{m_0}{F}\sinh\frac{F\tau}{m_0}, \tag{3.14}
$$

$\tau < 0$ and $\tau > 0$ correspond to $V_{0z} < 0$ and $V_{0z} > 0$, respectively. The response rate in this case is [7]

$$
\begin{aligned}
R \sim & \int dE \int dP_x \int dP_y \int d^3\mathbf{k} \mid A \mid^2 \\
\sim & \int d^3\mathbf{k} \frac{1}{\omega}\Big(\int d\tau e^{i\Delta\tau + i\omega t(\tau) - ik_z z(\tau)}\Big)^2 \\
\sim & \int d\tau \frac{\Delta m}{e^{2\pi\Delta m \cdot m_0/F} - 1}.
\end{aligned} \tag{3.15}
$$

This result indicates that the detector moving in the classical potential $-Fz$ responds as if it were immersed in a thermal bath. This is known as the Unruh effect, that is, the Unruh effect can be interpreted as bremsstrahlung by a heavy particle with two internal energy level.

4 summary and remark

In this paper we have suggested a transcription of a DeWitt detector. In our approach the detector is not forced on any trajectories. This enables to describe the flow on the energy. When it moves inertially, both the energy and the momentum conservation laws are automatically involved in its transition amplitude. This forbids the detector to respond in the Minkowski vacuum if $\Delta m \geq 0$. On the other hand, if the detector moving in the classical potential $-Fz$ is considered, momentum conservation in z direction is not demanded because the translational invariance is broken. Though energy conservation survives, it alone does not forbid the interaction mentioned above. In this context, the recoil of the detector is essential as Parentani pointed out [8]. Decrease in the detector's kinetic energy is origin of energy paid for the radiation and increase in the detector's rest mass.

The situation is rather similar to the case of rotating detectors though Bogoliubov coefficients between Minkowski and rotating modes have not been obtained [9, 10]. The rotating detector can be excited and emit a particle in the Minkowski vacuum, energy of which is supplied via the recoil of the detector [9]. Circumstances would almost be the same even if other classical potentials are considered. Therefore we can say the Unruh effect is not restricted to the case of uniformly acceleration but is a rather general phenomenon in classical potential though distribution of detectors' energy gaps would not be like a thermal bath.

Acknowledgments

The authors thank Dr. K. Ikegami for stimulating discussions.

References

[1] S. A. Fulling, Phys. Rev. **D7**, 2850 (1973)

[2] W. G. Unruh, Phys. Rev. **D14**, 870 (1976)

[3] P. C. W. Davies, J. Phys. **A8**, 609 (1975)

[4] B. S. DeWitt, in *General Relativity*: *An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge, 1979)

[5] W. G. Unruh and R. M. Wald, Phys. Rev. **D29**, 1047 (1984)

[6] S. Takagi, Prog. theor. Phys. Suppl. **88**, 1 (1986)

[7] N. D. Birrell and P. C. W. Davies, *Quantum fields in curved space* (Cambridge Univ. Press, Cambridge, 1982)

[8] R. Parentani, Nucl. Phys. **B454**, 227 (1995)

[9] T. Suga, R. Mochizuki and K. Ikegami, Prog. Theor. Phys. **101**, 783(1999)

[10] P. C. W. Davies, T. Dray and C. A. Manogue, Phys. Rev. **D53**, 4382 (1996)